



Functions and Graphs

HKDSE Mathematics Compulsory Part

Hoksi Education

Version 2026.1.0 | 2026-08-27

2 Functions and Graphs

2.1 Function notation

A **function** f assigns to each input x in its domain exactly one output $f(x)$. The set of all outputs is the **range**.

Example 2.1 (Evaluating a function). Let $f(x) = 2x^2 - 3x + 1$. Find $f(-2)$ and $f(a + 1)$.

Solution.

$$f(-2) = 2(-2)^2 - 3(-2) + 1 = 8 + 6 + 1 = 15,$$

$$f(a + 1) = 2(a + 1)^2 - 3(a + 1) + 1 = 2a^2 + a.$$

2.2 The graph of a quadratic function

The graph of $y = ax^2 + bx + c$ is a parabola. Completing the square gives the **vertex form**

$$y = a \left(x + \frac{b}{2a} \right)^2 + \frac{4ac - b^2}{4a},$$

so the vertex is at $x = -\frac{b}{2a}$ and the axis of symmetry is the vertical line through it.

- If $a > 0$ the parabola opens upwards and the vertex is a **minimum**.
- If $a < 0$ the parabola opens downwards and the vertex is a **maximum**.

Exam technique

Optimisation questions in the Compulsory Part almost always reduce to finding the vertex of a parabola. Complete the square rather than guessing.

Example 2.2 (Maximising an area). A rectangular pen is built against a straight wall using 40 m of fencing on the remaining three sides. Find the largest possible area.

Solution. Let the two sides perpendicular to the wall have length x m. Then the side parallel to the wall is $40 - 2x$ m, and the area is

$$A(x) = x(40 - 2x) = -2x^2 + 40x = -2(x - 10)^2 + 200.$$

Since the coefficient of x^2 is negative, A is greatest when $x = 10$, giving a maximum area of 200 m^2 .

2.3 Transformations of graphs

Starting from $y = f(x)$:

Table 2.1: Common graph transformations

Transformation	New equation	Effect
Translate up by k	$y = f(x) + k$	Shift k units in $+y$
Translate right h	$y = f(x - h)$	Shift h units in $+x$
Reflect in x -axis	$y = -f(x)$	Flip vertically
Reflect in y -axis	$y = f(-x)$	Flip horizontally
Vertical stretch	$y = kf(x)$	Scale y -values by k

2.4 Exercises

1. Let $f(x) = x^2 - 6x + 5$. Write $f(x)$ in vertex form and state the coordinates of the vertex.
2. The graph of $y = f(x)$ is translated 3 units to the left and 2 units down. Write the equation of the new graph in terms of f .
3. Find the maximum value of $-3x^2 + 12x - 7$.

Answers

1. $f(x) = (x - 3)^2 - 4$; vertex $(3, -4)$.
2. $y = f(x + 3) - 2$.
3. 5, attained at $x = 2$.

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