



學斯教育
HOKSI EDUCATION

Quadratic Equations in One Unknown

HKDSE Mathematics Compulsory Part

Hoksi Education

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1 Quadratic Equations in One Unknown

1.1 The discriminant

A quadratic equation in one unknown has the form

$$ax^2 + bx + c = 0, \quad a \neq 0, \quad (1.1)$$

where a , b and c are real. The quantity

$$\Delta = b^2 - 4ac$$

is called the **discriminant** of Equation 1.1, and it determines the nature of the roots.

Table 1.1: Nature of the roots of a quadratic equation

Discriminant	Nature of roots
$\Delta > 0$	Two distinct real roots
$\Delta = 0$	One double (repeated) real root
$\Delta < 0$	No real roots (two complex roots)

i Key result

The roots of Equation 1.1 are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

1.2 Worked example

Example 1.1 (Finding a range of values of k). The equation $x^2 + kx + (k + 3) = 0$ has two distinct real roots. Find the range of values of k .

Solution. Here $a = 1$, $b = k$ and $c = k + 3$, so

$$\Delta = k^2 - 4(1)(k + 3) = k^2 - 4k - 12 = (k - 6)(k + 2).$$

Two distinct real roots means $\Delta > 0$, so $(k - 6)(k + 2) > 0$. The product is positive when both factors are positive or both are negative, giving

$$k < -2 \quad \text{or} \quad k > 6.$$

1.3 Relations between roots and coefficients

If α and β are the roots of Equation 1.1, then

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}.$$

These identities let you answer questions about the roots without ever solving the equation.

Example 1.2 (Using the sum and product of roots). If α and β are the roots of $2x^2 - 5x + 1 = 0$, find $\alpha^2 + \beta^2$.

Solution. We have $\alpha + \beta = \frac{5}{2}$ and $\alpha\beta = \frac{1}{2}$. Then

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \left(\frac{5}{2}\right)^2 - 2\left(\frac{1}{2}\right) = \frac{25}{4} - 1 = \frac{21}{4}.$$

1.4 Exercises

1. Solve $3x^2 - 7x + 2 = 0$ by factorisation.
2. The equation $x^2 - 2mx + (m + 6) = 0$ has a double root. Find all possible values of m .
3. If α and β are the roots of $x^2 + 4x - 9 = 0$, find $\frac{1}{\alpha} + \frac{1}{\beta}$.
4. Form a quadratic equation whose roots are $3 + \sqrt{2}$ and $3 - \sqrt{2}$.

Answers

1. $x = \frac{1}{3}$ or $x = 2$.
2. $m = -2$ or $m = 3$.
3. $\frac{4}{9}$.
4. $x^2 - 6x + 7 = 0$.

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